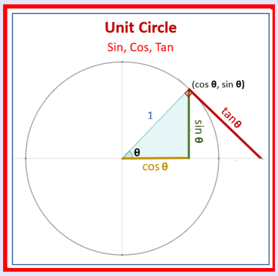
Math 241

Winter 2024

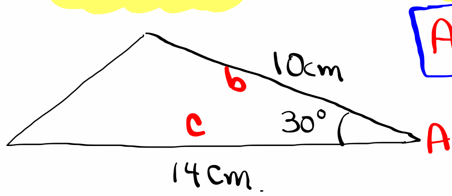
Lecture 6



Feb 19-8:47 AM

Class QZ 5

Find the area of the triangle below



$$\boxed{\text{Area} = \frac{1}{2}\, bc \,\text{Sin}\, A}$$

$$= \frac{1}{2}\cdot 10\cdot 14\cdot \text{Sin}\,30^\circ$$

$$= \frac{1}{2}\cdot 10\cdot 14\cdot \frac{1}{2} = \boxed{35\ \text{cm}^2}$$

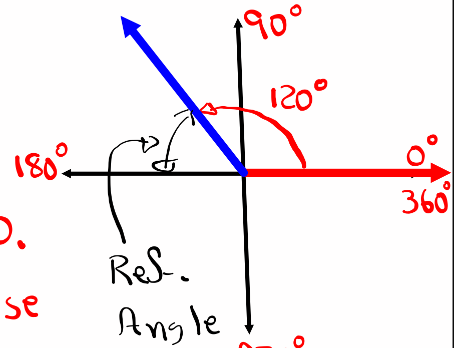
SAS

unit as cm ← Distance

unit as cm° ← Angle

Jan 9-12:16 PM

Draw 120° in standard position. Give its ref. angle in degrees.



Vertex at $(0,0)$

Initial Side on x-axis, $x>0$.

Angle >0 $\rightarrow$ counter clock wise

Angle <0 $\rightarrow$ Clockwise

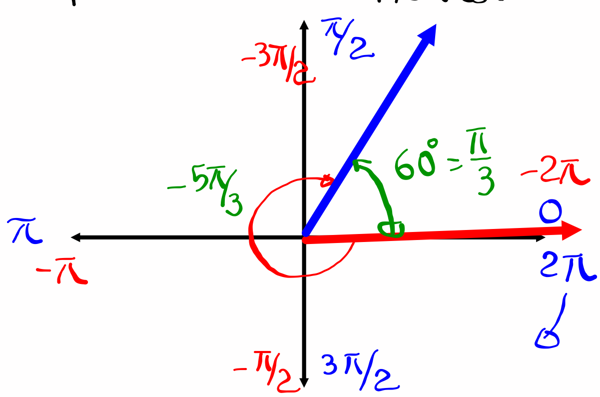
$180^\circ - 120^\circ = \boxed{60^\circ}$

Jan 10-8:04 AM

Draw $-\frac{5\pi}{3}$ in standard position. Find its ref. angle in radians.

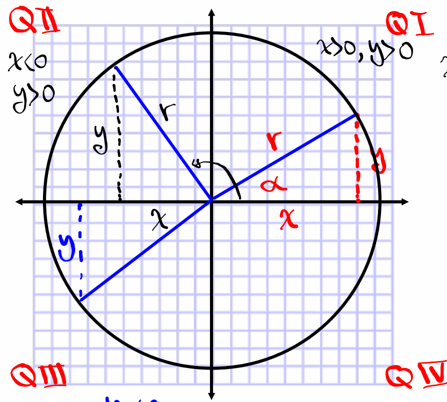
we should recognize that $\frac{\pi}{3} = 60^\circ$

$-\frac{5\pi}{3} = -5(60^\circ) = -300^\circ$



Ref. angle is $\frac{\pi}{3}$

Jan 10-8:08 AM



$x<0, y<0$

$\sin\alpha = \frac{y}{r} < 0$

$\cos\alpha = \frac{x}{r} < 0$

$\tan\alpha = \frac{y}{x} > 0$

$x^2 + y^2 = r^2$

$\sin\alpha = \frac{y}{r}$

$\cos\alpha = \frac{x}{r}$

$\tan\alpha = \frac{y}{x}$

QI

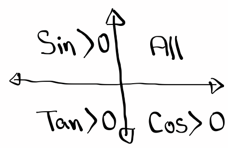
All are Positive

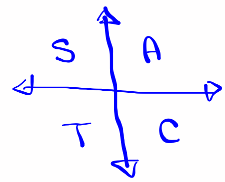
QII

$\sin\alpha = \frac{y}{r} > 0$

$\cos\alpha = \frac{x}{r} < 0$

$\tan\alpha = \frac{y}{x} < 0$

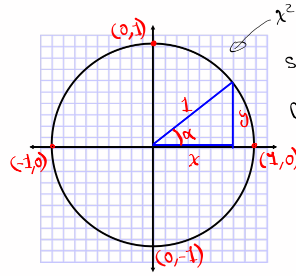




Jan 10-8:13 AM

Famous Angles:

$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π



$x^2 + y^2 = 1$

$\sin\alpha = \frac{y}{1} = y$

$\cos\alpha = \frac{x}{1} = x$

$\tan\alpha = \frac{y}{x}$

$\sin 0° = \sin 0 = 0$

$\cos 0° = \cos 0 = 1$

$\tan 0° = \tan 0 = \frac{0}{1} = 0$

$\sin 90° = \sin\frac{\pi}{2} = 1$

$\cos 90° = \cos\frac{\pi}{2} = 0$

$\tan 90° = \tan\frac{\pi}{2} = \frac{1}{0}$ undefined

$\sin 180° = \sin\pi = 0$

$\cos 180° = \cos\pi = -1$

$\tan 180° = \tan\pi = \frac{0}{-1} = 0$

$\sin 270° = \sin\frac{3\pi}{2} = -1$

$\cos 270° = \cos\frac{3\pi}{2} = 0$

$\tan 270° = \tan\frac{3\pi}{2} = \frac{-1}{0}$ undef.

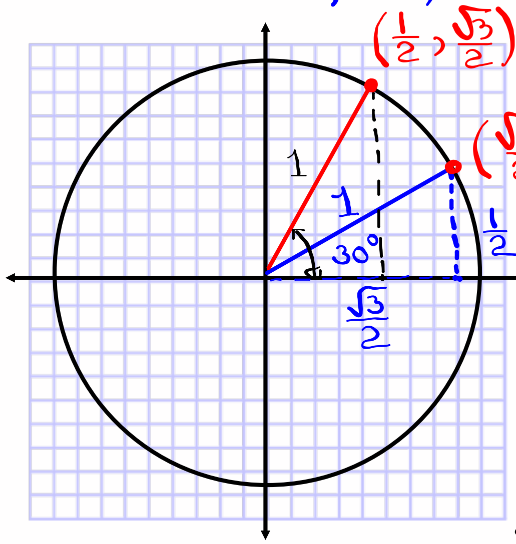
$\sin 360° = \sin 2\pi = 0$

$\cos 360° = \cos 2\pi = 1$

$\tan 360° = \tan 2\pi = \frac{0}{1} = 0$

Jan 10-8:22 AM

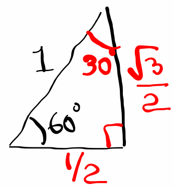
what about 30°, 60°, and 45°?



$\sin 30^\circ = \sin \frac{\pi}{6} = \frac{1}{2}$

$\cos 30^\circ = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$



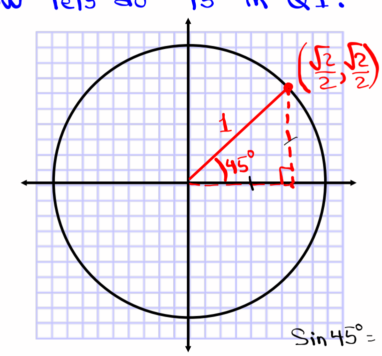
$\sin 60^\circ = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

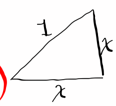
$\cos 60^\circ = \cos \frac{\pi}{3} = \frac{1}{2}$

$\tan 60^\circ = \tan \frac{\pi}{3} = \sqrt{3}$

Jan 10-8:34 AM

Now let's do 45° in QI:





$x^2 + x^2 = 1^2$

$2x^2 = 1$

$x^2 = \frac{1}{2} \qquad x = \sqrt{\frac{1}{2}}$

$x = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin 45^\circ = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos 45^\circ = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = \tan \frac{\pi}{4} = 1$

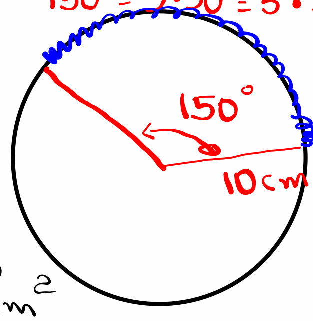
Degrees	0°	30°	45°	60°	90°	180°	270°	360°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und.	0	und.	0

Jan 10-8:41 AM

A circular Sector has a central angle of 150°

with radius 10 cm.

$150^\circ = 5 \cdot 30^\circ = 5 \cdot \frac{\pi}{6}$



1) find its area

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \cdot 10^2 \cdot \frac{5\pi}{6} = \boxed{\frac{125\pi}{3}} \text{ cm}^2$$

2) find its arc length.

$$S = r\,\theta = 10 \cdot \frac{5\pi}{6} = \boxed{\frac{25\pi}{3}} \text{ cm}$$

Jan 10-9:02 AM

Simplify

$$\sin^2\alpha\,(\underbrace{1 + \cot^2\alpha}_{\csc^2\alpha})$$

$$= \sin^2\alpha \cdot \csc^2\alpha = (\underbrace{\sin\alpha \cdot \csc\alpha}_{1})^2 = 1^2 = \boxed{1}$$

Verify

$$\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$$

$$\sec^4 x - \sec^2 x = \sec^2 x\,(\sec^2 x - 1)$$

$$= (1 + \tan^2 x)(\cancel{1} + \tan^2 x \cancel{-1})$$

$$= \tan^2 x\,(1 + \tan^2 x) = \tan^2 x + \tan^4 x$$

Jan 10-9:09 AM

Verify $\dfrac{1+\sin\theta}{1-\sin\theta} - \dfrac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta \cdot \sec\theta$

$$LHS = \frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} - \frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{1+2\sin\theta+\sin^2\theta - (1-2\sin\theta+\sin^2\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{\cancel{1}+2\sin\theta\cancel{+\sin^2\theta}\cancel{-1}+2\sin\theta\cancel{-\sin^2\theta}}{1-\sin^2\theta}$$

$$= \frac{4\sin\theta}{\cos^2\theta} = 4\cdot\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$$

$$= \boxed{4\tan\theta\ \sec\theta}$$

Jan 10-9:16 AM

Verify

$$(\sec\alpha - \tan\alpha)^2 = \frac{1-\sin\alpha}{1+\sin\alpha}$$

$$LHS = \left(\frac{1}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha}\right)^2$$

$$= \left(\frac{1-\sin\alpha}{\cos\alpha}\right)^2 = \frac{(1-\sin\alpha)^2}{\cos^2\alpha}$$

$$= \frac{(1-\sin\alpha)(1-\sin\alpha)}{1-\sin^2\alpha}$$

$$= \frac{\cancel{(1-\sin\alpha)}(1-\sin\alpha)}{\cancel{(1-\sin\alpha)}(1+\sin\alpha)}$$

$$= \boxed{\frac{1-\sin\alpha}{1+\sin\alpha}} \quad \text{RHS}$$

Recall

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

Jan 10-10:01 AM

Verify

$$\frac{\sin\theta}{1-\cos\theta} - \frac{\sin\theta\cos\theta}{1+\cos\theta} = \csc\theta(1+\cos^2\theta)$$

$$LHS = \frac{\sin\theta\,(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} - \frac{\sin\theta\cos\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} =$$

$$= \frac{\sin\theta(1+\cos\theta) - \sin\theta\cos\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{\sin\theta + \cancel{\sin\theta\cos\theta} - \cancel{\sin\theta\cos\theta} + \sin\theta\cos^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cancel{\sin\theta}(1+\cos^2\theta)}{\sin^{\cancel{2}}\theta} = \frac{1+\cos^2\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}\cdot(1+\cos^2\theta)$$

$$= \boxed{\csc\theta\,(1+\cos^2\theta)}$$

RHS

Jan 10-10:06 AM

Verify

$$\frac{1}{\tan x - \sec x} + \frac{1}{\tan x + \sec x} = -2\tan x$$

$$LHS = \frac{1(\tan x + \sec x) + 1(\tan x - \sec x)}{(\tan x - \sec x)(\tan x + \sec x)} =$$

$(A-B)(A+B)$
$= A^2 - B^2$

$$\frac{2\tan x}{\tan^2 x - \sec^2 x} =$$

Recall
$1 + \tan^2 x = \sec^2 x$
$\tan^2 x = \sec^2 x - 1$

$$\frac{2\tan x}{\cancel{\sec^2 x} - 1 - \cancel{\sec^2 x}} =$$

$$\frac{2\tan x}{-1} = \boxed{-2\tan x} \quad \text{RHS}$$

Jan 10-10:14 AM

Verify

$$\underbrace{\sin^3 x + \cos^3 x}_{A^3 + B^3} = \underbrace{(\sin x + \cos x)(1 - \sin x \cos x)}_{\text{factored}}$$

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

$$\sin^3 x + \cos^3 x = (\sin x + \cos x)(\boxed{\sin^2 x} - \sin x \cos x \boxed{+ \cos^2 x})$$

$$= \boxed{(\sin x + \cos x)(1 - \sin x \cos x)}$$

RHS

Jan 10-10:21 AM

Verify

Recall

$$(A-B)^2 = A^2 - 2AB + B^2$$

$$\frac{(\sec\theta - \tan\theta)^2 + 1}{\sec\theta \csc\theta - \tan\theta \csc\theta} = 2\tan\theta$$

$\sec^2\theta$

$$LHS = \frac{\sec^2\theta - 2\sec\theta\tan\theta + \boxed{\tan^2\theta + 1}}{\csc\theta(\sec\theta - \tan\theta)}$$

$$= \frac{2\sec^2\theta - 2\sec\theta\tan\theta}{\csc\theta(\sec\theta - \tan\theta)}$$

$$= \frac{2\sec\theta\,\cancel{(\sec\theta - \tan\theta)}}{\csc\theta\,\cancel{(\sec\theta - \tan\theta)}} = 2 \cdot \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}$$

$$= 2 \cdot \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} = \frac{2\sin\theta}{\cos\theta} = \boxed{2\tan\theta} \quad \text{RHS}$$

Jan 10-10:27 AM

Simplify

$\sin^2 x\,(1+\cot x)+\cos^2 x\,(1-\tan x)+\cot^2 x$

$= \sin^2 x + \sin^2 x\cdot\cot x + \cos^2 x - \cos^2 x\cdot\tan x + \cot^2 x$

$= 1 + \sin^{\cancel{2}} x\cdot\dfrac{\cos x}{\cancel{\sin x}} - \cos^{\cancel{2}} x\cdot\dfrac{\sin x}{\cancel{\cos x}} + \cot^2 x$

$= \csc^2 x + \cancel{\sin x\cos x} - \cancel{\cos x\sin x}$

$= \boxed{\csc^2 x}$

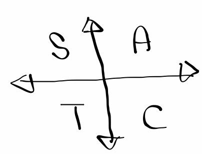
Jan 10-10:36 AM

How to find values of trig. functions in QII, QIII, or QIV:

1) Find the ref. angle

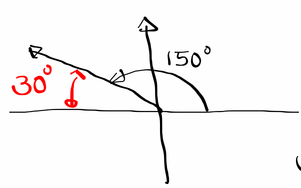
2) Find the value of trig. function of the ref. angle

3) Determine its sign



ex: find $\sin 150^\circ = +\sin 30^\circ$

$= \boxed{\frac{1}{2}}$



Now

$\cos 150^\circ = -\cos 30^\circ$

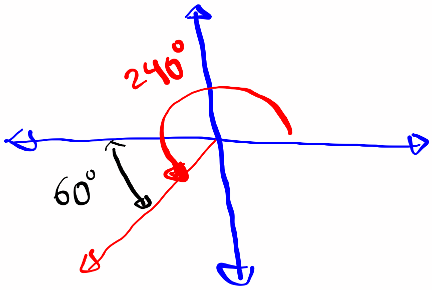
$= \boxed{-\frac{\sqrt{3}}{2}}$

what about

$\tan 150^\circ = -\tan 30^\circ = \boxed{-\frac{\sqrt{3}}{3}}$

Jan 10-10:43 AM

Find $\sin 240^\circ$



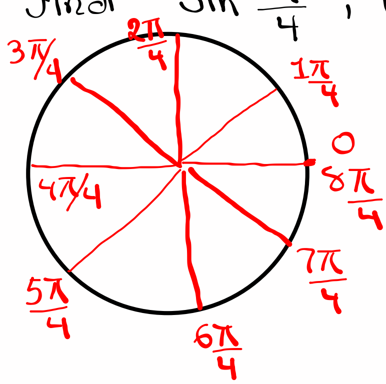
$\sin 240^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

$\cos 240^\circ = -\cos 60^\circ = -\frac{1}{2}$

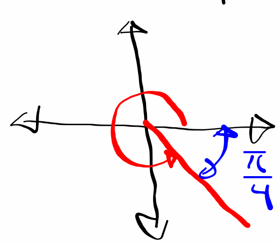
$\tan 240^\circ = +\tan 60^\circ = \sqrt{3}$

Jan 10-10:49 AM

Find $\sin\frac{7\pi}{4}$, $\cos\frac{7\pi}{4}$, and $\tan\frac{7\pi}{4}$.



Recall $\frac{\pi}{4} = 45^\circ$



$\sin\frac{7\pi}{4} = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

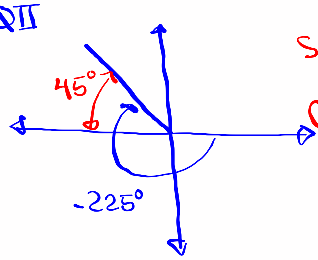
$\cos\frac{7\pi}{4} = +\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\tan\frac{7\pi}{4} = -\tan\pi/4 = -1$

Jan 10-10:52 AM

Find $\sin(-225)^\circ$, $\cos(-225)^\circ$, and $\tan(-225)^\circ$.

QII

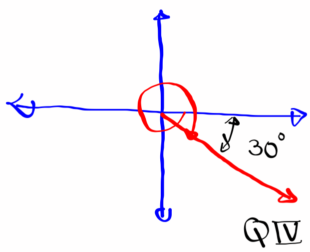


$\sin(-225)^\circ = +\sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos(-225)^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

$\tan(-225^\circ) = -\tan 45^\circ = -1$

Find the value of all six trig functions for angle -390°.



$\sin -390^\circ = -\frac{1}{2}$ $\qquad \csc -390^\circ = -2$

$\cos -390^\circ = +\frac{\sqrt{3}}{2}$ $\qquad \sec -390^\circ = \frac{2\sqrt{3}}{3}$

$\tan -390^\circ = -\frac{\sqrt{3}}{3}$ $\qquad \cot -390^\circ = -\sqrt{3}$

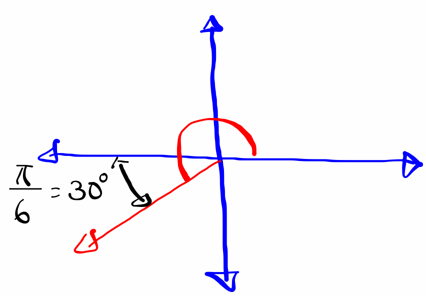
Jan 10-10:57 AM

Find values of all six trig functions of angle $\frac{7\pi}{6}$.

$\frac{\pi}{6} = 30^\circ$

$\frac{7\pi}{6} = 7(30^\circ) = 210^\circ$

we are in QIII



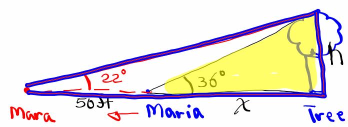
$\sin \frac{7\pi}{6} = -\frac{1}{2}$ $\qquad \csc \frac{7\pi}{6} = -2$

$\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ $\qquad \sec \frac{7\pi}{6} = -\frac{2\sqrt{3}}{3}$

$\tan \frac{7\pi}{6} = +\frac{\sqrt{3}}{3}$ $\qquad \cot \frac{7\pi}{6} = \sqrt{3}$

Jan 10-11:07 AM

Maria's angle of elevation to the top of a tree is 36°. She moves 50 ft away from the tree, her angle of elevation to the top of the tree is now 22°. Find the height of tree, rounded to whole feet. Complete drawing required.



$\tan 36° = \frac{h}{x}$

$\tan 22° = \frac{h}{x+50}$

$h = x \tan 36°$

$h = (x+50) \cdot \tan 22°$

$\Rightarrow x \tan 36° = (x+50) \cdot \tan 22°$

$x \tan 36° = x \tan 22° + 50 \cdot \tan 22°$

$x \tan 36° - x \tan 22° = 50 \tan 22°$

$x \boxed{(\tan 36° - \tan 22°)} = 50 \tan 22°$

$x = \frac{50 \tan 22°}{\tan 36° - \tan 22°}$

$h = \frac{50 \cdot \tan 22°}{\tan 36° - \tan 22°} \cdot \tan 36°$

$h = \frac{50 \tan 22° \cdot \tan 36°}{\tan 36° - \tan 22°}$

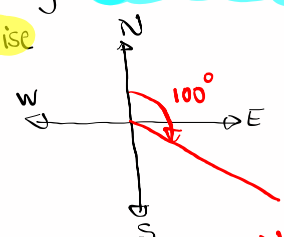
$h \approx 45.508 \approx \boxed{46 \text{ ft}}$

Jan 10-11:14 AM

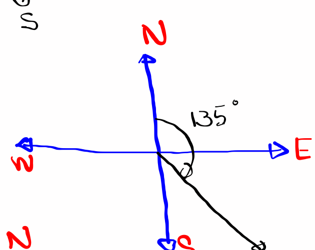
Intro. to bearing

Method I: Measuring from North axis, going clockwise

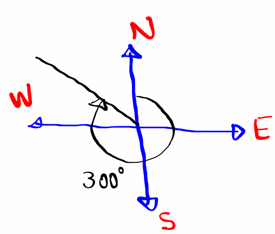
ex: Bearing of 100°



ex. Bearing of 135°



ex: Bearing of 300°

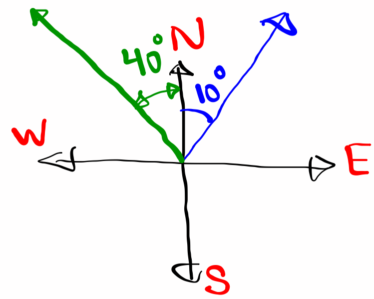


Jan 10-11:32 AM

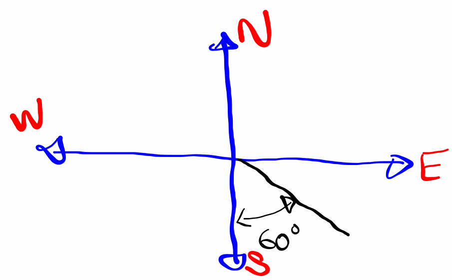
Method II: Measuring from North and South axis in the direction of East and West.

ex: N 10° E

ex: N 40° W



ex: S 60° E



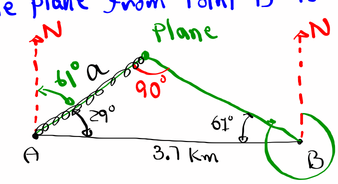
Jan 10-11:37 AM

Draw as you read

Points A and B are on East–West line and 3.7 Km apart

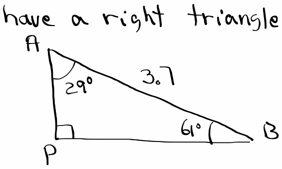
Bearing of a plane from Point A is 61°.

Bearing of the plane from Point B is 331°.



331° − 270° = 61°

We have a right triangle



How far is the plane from both radar Stations A & B ?

$\sin 61^\circ = \frac{a}{3.7}$

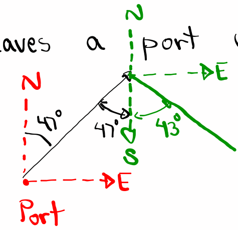
$a = 3.7 \cdot \sin 61^\circ$

$\boxed{a \approx 3.2 \text{ Km}}$

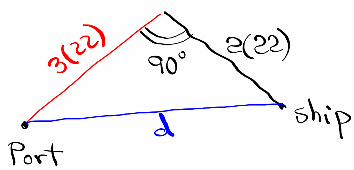
Jan 10-11:42 AM

A ship has a speed of 22 knots (22 mph).

It leaves a port with a bearing of N 47° E. then it turns and has a bearing of S 43° E.



Suppose the ship sailed for $\boxed{3\text{hrs}}$ before changing direction and it sailed 2 hrs on new bearing. How far is the ship from the original port?



$d^2 = 66^2 + 44^2$

$= 6292$

$d = \sqrt{6292}$

$= 79.322$

$\approx$ 79 miles

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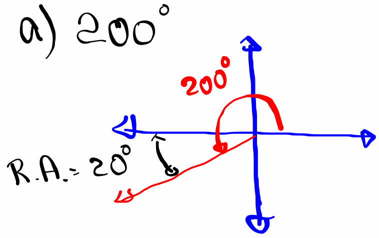
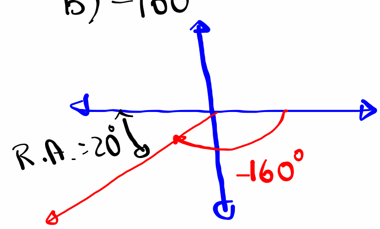
Class QZ 6

Draw angles in standard position, find its ref. angle

a) 200°



b) −160°



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